Dr.V.S.KRISHNA GOVT.DEGREE COLLEGE (A), VISAKHAPATNAM.

CBCS/ SEMESTER SYSTEM

(w.e.f. 2022-23)

B.A./B.Sc. MATHEMATICS

COURSE-III, ABSTRACT ALGEBRA

MODEL PAPER

Time:3Hrs **Max.Marks:75M**

SECTION - A

Answer any FIVE_questions. Each question carries FIVE marks 5 X 5 M=25 M

1. Prove that the set Q^1 of rational numbers other than 1 forms an abelian group w.r.to the operation * defined by a*b =a+b-ab for all $a,b \in Q^1$.

2. If H is a subgroup of a group G then prove that, for $a,b \in G$, $a \in Hb \Leftrightarrow Ha = Hb$
3. Define order of an element. In a group G, prove that if $a \in G$ then $o(a) = o(a^{-1})$.

4. Show that every subgroup of a group G with index 2 is a normal subgroup of the group G.

5. Find the regular permutation group isomorphic the multiplicative group $\{1, \omega, \omega^2\}$

6. If f is a homomorphism of a group G into a group G' then prove that kernel f is a normal subgroup of G.
7. Show that field has no proper ideals.
8. Define a Boolean ring and prove that in a Boolean ring R, $a + a = 0 \forall a \in R$

SECTION-B

Answer any FIVE questions. Each question carries TEN marks. 5 X 10 M=50 M

9. a) Prove that a finite semi group satisfying the cancellation laws is a group

(or)

b) Prove that the set of matrices $A_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$; $\alpha \in R$ forms an abelian group w.r.to the matrix multiplication, if $\cos\theta = \cos\phi \Rightarrow \theta = \phi$

10. a). If H and K are two subgroups of a group G, then show that $H \cup K$ is a subgroup of G iff either $H \subseteq K$ or $K \subseteq G$.

(or)

b). State and prove Lagrange's theorem for finite groups.

11 a). Let G and $G^{'}$ be two groups . If $f:G\rightarrow G^{'}$ is a homomorphism then , prove the following

i) $f(e)=e^{'}$ where e and $e^{'}$ are the identity elements in

ii) $f(a^{-1})=[f(a)]^{-1}$, for each $a\in G$.

(or)

b). State and prove fundamental theorem of homomorphism of groups

12. a). State and prove cayley's theorem.

(or)

b). Show that every subgroup of a cyclic group is cyclic

13. a). Prove that every finite integral domain is a field.

(or)

b). Show that the ring of integers Z is a principal ideal ring.